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ELASTICITY AND VISCOSITY OF CARROT ROOT TISSUE AT DIFFERENT RATE OF DEFORMATION

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Viscosity of foods and their mechanical properties are essential for modeling the mechanical behaviour of foods during deformation and flow. The rheological models are used for engineering calculations, *e.g*., during design of food processing machines, pumping systems, packaging machines, *etc*. Although the classical linear rheology is a rate-independent theory, the mechanical properties of the viscoelastic plant materials depend on a deformation rate. The influence of a deformation rate on mechanical properties of biological viscoelastic materials needs to be documented and it is the aim of this paper.

It is proved, based on the Maxwell model, that the modulus of elasticity and the viscosity ratio are decreasing functions of the rate of deformation. Next, the modulus of elasticity and the viscosity ratio of the carrot cylindrical samples are determined using discrete-time measurements of the reaction force obtained in the compression and relaxation tests at a wide range of preliminary deformation rates, *i.e.* from 1.67×10^{-4} m·s^{-1} to 1.5 m·s^{-1}. The relaxation data were processed using a four-element Maxwell model and the changes in viscoelastic constants were determined as a function of the deformation rate. The stress relaxation both in the state of uniaxial stress as well as in the state of uniaxial strain is considered. The experimental results are in good agreement with theoretical analysis for the biological material considered. The results motivate hypothesis that the number of microcrackings increases along with the deformation rate and indicate the irreversible character of the changes that proceed in plant materials under loading.

INTRODUCTION

The rheological properties of plant materials like, fruits (apple, pear) root vegetables (carrot, potato) and some industrial plants (sugar beet) have been described for tens of years by the linear constitutive models that at a small deformation provide satisfactory approximation of their mechanical properties [Rao, 1999]. In the classical theory of linear rheological models, however, their parameters are independent of a deformation speed, while in practice the irreversible character of the changes occurring in the agricultural materials due to deformation or loading shows that this assumption is a too far fetched idealization of the reality. This fact, among others, is confirmed by the results of experiments on the adequacy of linear viscoelastic models presented in the paper [Gołacki & Stropek, 2004]. The basic source of information on the viscoelastic properties of biological materials appears to be elasticity moduli and viscosity ratios as they made possible to analyse in detail behaviour of the fruit and vegetable subjected to various deformations and loading using the finite elements or the boundary elements method. A classical, especially for plant materials, manner of studying viscoelasticity is by two-phase stress relaxation test. In the first initial phase the stress should be imposed instantaneously, while during the second one the corresponding force induced in the specimen is measured.

The objective of the present work was to study the influence of a deformation rate on the properties of material modified during the first phase of the relaxation test, that is during the preliminary deformation.

MATHEMATICAL BACKGROUND

The Maxwell model of the form [Blahovec, 2001; Chen, 1994; Rao, 1999]:

$$\frac{1}{E}\dot{\sigma} + \frac{1}{\eta}\sigma = \dot{\varepsilon} \tag{1}$$

where η is the viscosity ratio and E denotes the modulus of elasticity, is the commonly used way of description of the relationship between the stress σ and strain ε in plant linear viscoelastic materials in the range of small deformations. In the above equation $\dot{\sigma}$ and $\dot{\varepsilon}$ denote, respectively, the stress and strain derivatives with respect to the time t, *i.e.*, $\dot{\sigma}=d\sigma/dt$ and $\dot{\varepsilon}=d\varepsilon/dt$. The equation (1) can be rewritten in the form:

$$\dot{\sigma} + \frac{E}{\eta}\sigma = E\dot{\varepsilon} \tag{2}$$

To analyse the effect of a deformation speed $\dot{\varepsilon}$ on elasticity modulus E, the equation (2) should be differentiated on both sides with respect to E and $\dot{\varepsilon}$. Thus, we obtain:

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$$\frac{1}{\eta}\sigma\, dE = \dot{\varepsilon}\, dE + E\, d\dot{\varepsilon} \tag{3}$$

where dE denotes the elasticity modulus differential, while $d\dot{\varepsilon}$ is the deformation rate differential. Hence, taking account of (1), we obtain the following relation:

$$\frac{\mathrm{d}E}{\mathrm{d}\dot{\varepsilon}} = \frac{-\dot{\sigma}}{\left(\dot{\varepsilon} - \frac{\sigma}{\eta}\right)^2} \tag{4}$$

In the first phase of relaxation test, the stress σ increases along with time, so $\dot{\sigma}>0$ and on the basis of the equation (4) we conclude that $dE/d\dot{\varepsilon}<0$. It means that the elasticity modulus E decreases with a growing deformation rate $\dot{\varepsilon}$. Since, taking account of the equation (1), if $\dot{\sigma}>0$ then inequality $\dot{\varepsilon}-\sigma/\eta>0$ holds from the formula (4) it is clear that the influence of a deformation rate $\dot{\varepsilon}$ on the elasticity modulus E is less with a higher speed value.

To study the influence of a deformation rate $\dot{\varepsilon}$ on viscosity ratio η we will use the following equation arising directly from the equation (1):

$$\frac{\eta}{E}\dot{\sigma} + \sigma = \eta\,\dot{\varepsilon} \tag{5}$$

Differentiating the equation (5) on both sides with respect to η and $\dot{\varepsilon}$ we obtain the following equation:

$$\frac{1}{E}\dot{\sigma}\, d\eta = \eta\, d\dot{\varepsilon} + \dot{\varepsilon}\, d\eta \tag{6}$$

where $d\eta$ denotes the viscosity ratio differential. Then the derivative $d\eta/d\dot{\varepsilon}$ can be expressed as:

$$\frac{\mathrm{d}\eta}{\mathrm{d}\dot{\varepsilon}} = \frac{\eta}{\left(\frac{\dot{\sigma}}{\mathrm{E}} - \dot{\varepsilon}\right)}$$

Hence, taking account of (1) after simple manipulations we obtain:

$$\frac{\mathrm{d}\eta}{\mathrm{d}\dot{\varepsilon}} = \frac{-\sigma}{\left(\dot{\varepsilon} - \frac{\dot{\sigma}}{\mathrm{E}}\right)^2} \tag{7}$$

Since $\sigma>0$, by virtue of (7), the derivative $d\eta/d\dot{\varepsilon}<0$.

Thus, the viscosity ratio η also decreases along with the growing deformation rate $\dot{\varepsilon}$. In view of the equation (1) the inequality $\dot{\varepsilon}-\dot{\sigma}/E>0$ holds. Whence, on the basis of (7) we also conclude that the influence of a deformation speed on the viscosity ratio η is higher if the deformation rate $\dot{\varepsilon}$ is lower.

MATERIAL AND METHODS

The experimental object was constituted by carrot Perfekcja variety. From each root the cylindrical samples 20 mm in diameter and height were cut out and subjected to the stress relaxation test with the initial axial compression by 1 mm value.

The experiment was performed in both loading conditions, the state of uniaxial stress and the uniaxial strain state. The specimens examined in the uniaxial stress state underwent deformation between two parallel plates. The samples investigated in the state of uniaxial strain were placed in the cylinder and deformed by the travelling piston to prevent side deformation.

The relaxation tests at the quasi-static loading conditions were performed in the universal testing machine INSTRON 6022, while the measurements at the impact load conditions on the stand for dynamic testing [Gołacki *et al*., 1999]. The samples underwent the preliminary deformation at a constant speed. Having obtained the required value of deformation, a declining response force of a specimen over 120 s was recorded.

The experimental studies were performed for the following six different rates of initial deformation: 0.16 mm·s^{-1}, 0.83 mm·s^{-1}, 3.33 mm·s^{-1} and 0.5 m·s^{-1},1 m·s^{-1}, 1.5 m·s^{-1}. The first three values correspond to the quasi-static load conditions and the other three to impact loads.

On the basis of the obtained discrete measurements of the sample response force and using the algorithm presented in detail in the paper [Stankiewicz & Gołacki, 2004] there were determined the moduli of elasticity E_{ij} and viscosity ratios η_{ij}, i=1.2, j=1.2, in the four-parameter Maxwell models describing the relaxation function in the uniaxial stress state:

$$\Phi(t) = E_{11}\exp\left[-\left(\frac{E_{11}}{\eta_{11}}\right)t\right] + E_{12}\exp\left[-\left(\frac{E_{12}}{\eta_{12}}\right)t\right] \tag{9}$$

and the relaxation function in the state of uniaxial strain:

$$X(t) = E_{21}\exp\left[-\left(\frac{E_{21}}{\eta_{21}}\right)t\right] + E_{22}\exp\left[-\left(\frac{E_{22}}{\eta_{22}}\right)t\right] \tag{10}$$

It should be emphasized that the algorithm presented in the paper by Stankiewicz & Gołacki [2004] makes use of a formula given by Chen & Fridley [1972] depicting the course of cylindrical specimen reaction force in the case when stress relaxation process is described by the four-parameter Maxwell model. This formula evidently includes the deformation rate applied in the initial phase of the relaxation test. As a consequence, a deformation speed is allowed to be included into the algorithm of the best model of the forms (9) and (10) determination.

RESULTS AND DISCUSSION

Figures 1-4 present the measured dependence of the elasticity moduli E_{ij} and viscosity ratios η_{ij} *versus* deformation rate for carrot Perfekcja variety samples investigated in the state of uniaxial stress and uniaxial strain. To increase the diagrams legibility owing to a wide range of the examined speeds, the rate-axis is provided with the logarithmic scale.

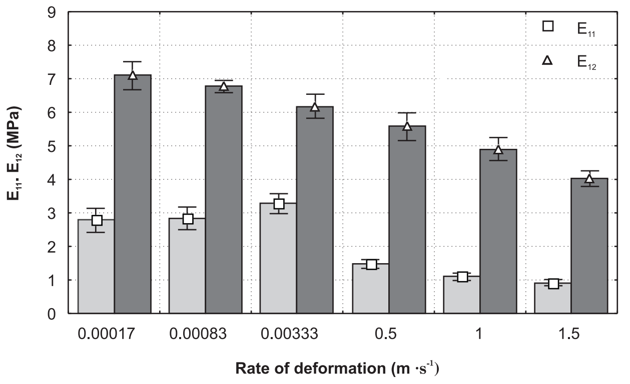

FIGURE 1. Modulus of elasticity E_{11} and E_{12} *versus* deformation rate of the carrot samples in the state of uniaxial stress.

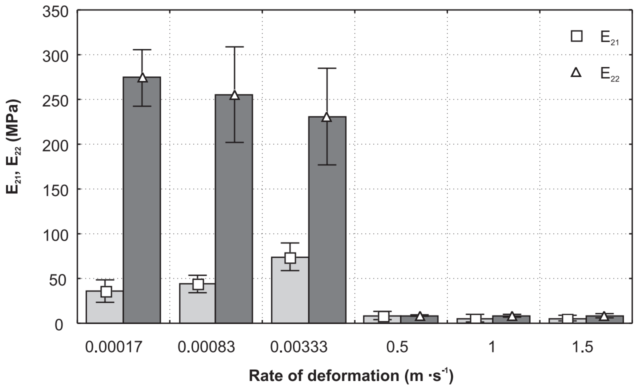

FIGURE 3. Modulus of elasticity E_{21} and E_{22} *versus* deformation rate of the carrot samples in the state of uniaxial strain.

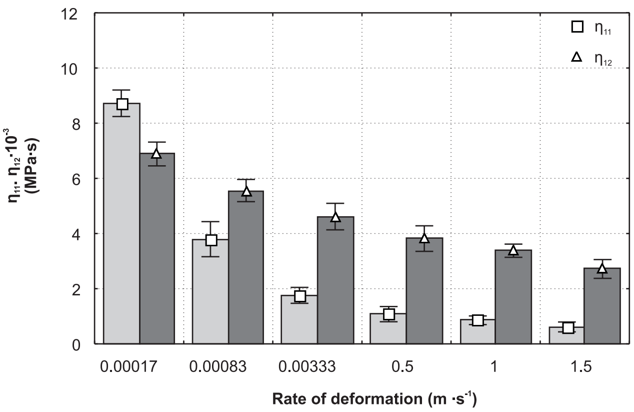

FIGURE 2. Viscosity ratios η_{11} and η_{12} *versus* deformation rate of the carrot samples in the state of uniaxial stress.

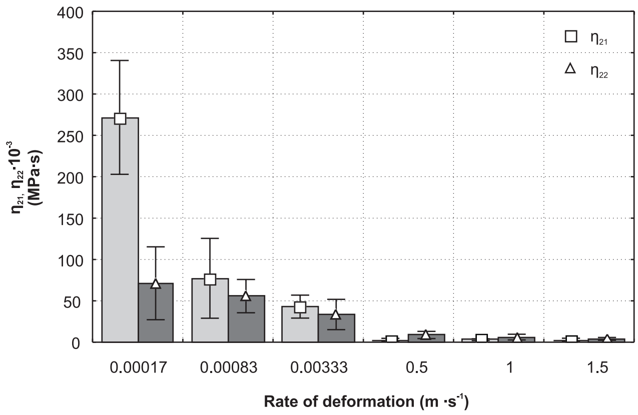

FIGURE 4. Viscosity ratios η_{21} and η_{22} *versus* deformation rate of the carrot samples in the state of uniaxial strain.

The experimentally obtained relationships of elasticity moduli E_{ij} values and viscosity ratios η_{ij} confirm the results of the theoretical analysis. The values of E_{ij} and η_{ij} parameters decline along with a deformation rate rise. The specimens examined in the state of the uniaxial stress showed a higher dependence of the elasticity moduli on a deformation rate compared to the samples subjected to the uniaxial strain. A similar relation was recorded analysing the effect of a deformation rate on viscosity ratios.

At high deformation rates, the cellular structure of biological materials undergoes a substantial degradation as demonstrated by the lower values of both, elasticity moduli E_{ij} and viscosity ratios η_{ij}.

The indicated relationships arise from the irreversible character of the processes occurring in the plant material tissues and are associated with the cell walls ruptures, structure delamination as well as the effects of the cell fluid filtration and its pumping over. Thus, the hypothesis concerning an increase in the number of microcrackings in plant material tissues along with a deformation rate rise is confirmed. The results of investigations can be used to determine a degradation stage of the internal tissue lattice. It is probable that a more detailed analysis on the influence of a deformation rate on the viscoelastic properties of the agricultural materials can be performed basing on the theory of the quasi-linear models [Fung, 1981].

CONCLUSIONS

1. There was determined the effect of the deformation rate on the elasticity moduli and dynamic viscosity ratios of the viscoelastic materials described by the Maxwell model.

2. The experimental studies on the carrot samples subjected to the stress-relaxation test in a wide range of the initial rates of deformation, compressed freely and in cylinders, have confirmed that the elasticity moduli and viscosity ratios decrease along with the deformation rate growth.

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WYBRANE LEPKOSPRĘŻYSTE CHARAKTERYSTYKI KORZENIA MARCHWI W RÓŻNYCH WARUNKACH OBCIĄŻEŃ

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W artykule przeprowadzono analizę teoretyczną zależności pomiędzy parametrami mechanicznymi charakterystyk materiałów o własnościach lepkosprężystych, a prędkością deformacji wstępnej. Eksperyment polegał na przeprowadzeniu testów ściskania i relaksacji naprężeń na walcowych próbkach marchwi w stanie jednoosiowego naprężenia i jednoosiowego odkształcenia. Stwierdzono spadek wartości współczynników sprężystości i lepkości wraz ze wzrostem prędkości deformacji, co świadczy o nieodwracalnym charakterze procesów zachodzących w materiale roślinnym w wyniku przyłożonego obciążenia.