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POSSIBILITIES OF APPLYING PARAMETRIC TESTS IN PRODUCTION CONTROL

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The study was aimed at developing, in the spreadsheet environment, an application for the assessment of product batch compliance with values provided in product's specification, by applying parametric tests method.

The investigation was carried out on batches of selected products consisting of 300 pieces of packed product. Dixon's Q test was applied for verification of the data obtained aiming at elimination of data burdened with gross error. Significance t test was applied for verification of statistical hypotheses where the null hypothesis H_0: $x_{śr} > m$ was verified against the alternative hypothesis H_1: $x_{śr} \leq m$.

In the tests applied, the inference is based on confidence interval and depends on the sample size.

The sample size influences the size of confidence interval and, as a consequence, the precision and reliability of inference. The software developed allows analysis aiming at determining at what minimum sample size further testing aimed at determining compliance of the product with a standard can be abandoned. Determination of a specific limit of deviations from the value declared by the producer in product's specification is particularly important for enterprises that have not yet included the allowed limit at which the product is still considered compliant with the standard in their enterprise standards. The application developed also allows the assessment of financial losses incurred by the enterprise as a consequence of inappropriate control of product quantity compliance with the declared value.

The analysis carried out may serve as an additional component in management allowing simple calculation of the losses incurred as a consequence of incorrect mass or volume of product per unit.

INTRODUCTION

The last decade has brought an unmatched acceleration in methodologies of approaches to quality issues. Quality has become one of the major factors determining market success of companies. It is understood in different ways; most frequently it is linked to products' usability or the degree to which they can satisfy the needs of buyers. The earlier approaches of considering client expectations, products' reliability or timeliness of deliveries are currently treated as standards that must be complied with. Educational processes provided the clients with specific knowledge that gave shape to their expectations [Strużyński, 2001]. Acceptance of products by clients is important, however assurance that the product complies with specifications provided is equally important [Paddmaja *et al.*, 2001].

In some companies quality control is carried out in a highly simplified way. The results obtained from tests on random selected samples are compared with the requirements provided in product's specification. That approach is erroneous and reflects the false results of quality control.

Credibility of random sample testing results depends on the degree to which that sample is representative for the total population and probability calculus allows assessment of credibility of conclusions concerning the entire population [Sobczyk, 2004].

Statistical quality control represents testing a sample collected from the controlled batch (general population) followed by analysis of the results obtained by formulating hypotheses and verification of them The result of random sample testing is random as it depends on which items are picked up from the set to form the sample and as a consequence it may differ from the result of testing the entire population of products it represents. That influences the method of test results statistical analysis and requires care in formulating conclusions concerning the population of products represented by the sample [Koronacki, 1999].

The majority of hypotheses verification methods are based on distributions of statistics χ^2, Snedecor's-F and Student's-t tests.

Selection of adequate statistical method and determination of its distribution assuming that verified hypothesis H_0 is true forms the basic issue in designing a statistical test. When such distribution is known, a certain very small probability α referred to as the significance level is selected. Next among the allowed values of the statistics a critical set ω is determined where the probability that the statistics will assume the value that belongs to the set ω is not higher than α. A sample is then collected and the value of statistics for it is computed. If the calculated value of that statistics belongs to the critical set ω, the verified hypothesis should be rejected. The probability of making an error represent-

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ing rejection of a true hypothesis is equal to the assumed significance level α. If it is established that the value of statistics for the set does not belong to the critical set ω, than it is decided that the result obtained from the sample does not contradict the hypothesis, which, however, does not prove that the hypothesis is true [Sobczyk, 2004].

The objective of this study was to develop, in the spreadsheet environment, an application for the assessment of product batch compliance with values provided in product's specification, by applying parametric tests method.

MATERIAL AND METHODS

The data for designing statistical tests was collected at a selected Dairy Cooperative using the following products that were characterized by different mass and different packaging for the study: (1) drinking milk with 2% fat content packed in 1 L bags made of polyethylene foil, and (2) homogenised cottage cheese packed in 150 g pressed boxes of polystyrene closed with a welded lid of PE/AL laminate.

During the final production process step – packaging – batches of 300 packages each of products to be examined were collected. Next, each package of the product batches selected was weighed.

The net mass of individual products for each batch of products tested was obtained by collecting 35 empty packages of each type that were weighed. Their average weight was calculated, which was then deducted from the gross mass of each product package. Products were weighed as follows: milk – on Model 15 MX AGROMERC scales with accuracy of d=0.001 kg, homogenised cheese and empty unit packages – on type B 2000 AXIS scales with accuracy of d=0.1 g.

Milk mass was converted to dm^3 using the determined milk density of 1.0333 g/cm^3. All measurements were taken at packaging temperature of 5°C.

Verification of data obtained was carried out using Dixon's Q test, based on spread distribution in the sample, to eliminate data burdened with gross errors. The significance t test was applied as the method for verification of the statistical hypotheses: the null hypothesis H_0: $x_{śr}>m$ and the alternative hypothesis H_1: $x_{śr} \leq m$.

The m values were assumed as follows: in the case of drinking milk – the capacity declared by the producer, *i.e.* m=1 dm^3, whereas in the case of homogenised cottage cheese – its net weight, *i.e.* m=150 g.

RESULTS AND DISCUSSION

The work spreadsheet of the application developed is presented in Figure 1. The cells with white background are used by the user for input of data necessary for analyses: (1) the sure value m, which may represent the standard value for the investigated characteristic, or the suggested value; (2) significance level α as the limit of allowed risk of errors in inference, and (3) description of the investigated characteristic in the product.

Spreadsheet cells with grey background contain calculation and logic formulas. They have been additionally protected against changing, which protects the software against corrupting. The application prepared in this way is a universal tool that can be used for control of compliance for any investigated characteristic digitally described against the declared value determined as the allowed minimum.

Unilateral Student t-test was applied for assessment of individual parameters investigated in 300 samples (n=300).

In Figure 1, the declared value of m – 1 dm^3 was given as the first sure value. Next, to present in more detail the sensitivity of that tool, the value of 1.005076 dm^3 was determined which represents the limit of compliance with the real value at the average product capacity of 1.0068 dm^3.

Determining the consecutive values as sure m the range within which the average does not comply with the requirements was determined precisely. The last value – 1.068162 dm^3 represents the limit exceeding which represents noncompliance with the formulated hypothesis.

Formulation of null hypothesis that should be rejected is the key to inference in that type of test. Inference is based on the alternative hypothesis, *i.e.* Student's t significance test the "average is equal or higher than m".

H_0: $x_{śr}<m$	The average value is lower than m				
H_1: $x_{śr} \geq m$	The average is equal or higher than m				
					α=0.05
Sure value m=	1.000	1.005076	1.005077	1.00681615	1.0068162
Symbol	MLEKO 2%	MLEKO 2%	MLEKO 2%	MLEKO 2%	MLEKO 2%
$x_{śr}$=	1.0068	1.0068	1.0068	1.0068	1.0068
$l_{x(1-\alpha)}=\pm$	0.0021	0.0021	0.0021	0.0021	0.0021
s(x) =	0.0183	0.0183	0.0183	0.0183	0.0183
$s^2(x)$ =	0.0003	0.0003	0.0003	0.0003	0.0003
N =	300	300	300	300	300
Compared	1 with m	2 with m	3 with m	4 with m	5 with m
$t_{obl.}$ =	6.4638	1.6502	1.6493	0.000026	-0.000022
P(t) =	0.0000000002	0.0500	0.0501	0.5000	
$t_{\alpha;f}$ =	1.6500	1.6500	1.6500	1.6500	
H_0:	Reject	Reject	Accept	Accept	Error
Change H_0 to:					x≥m

FIGURE 1. A spreadsheet of unilateral Student t-test.

Defining the sure value with a number of decimal places allows showing the sensitivity of the test. The precision is determined according to the user's needs. As shown in the figure, the delay in software reaction to changes in values is close to zero and the reaction in inference is immediate.

Based on the same principles the application was developed for the null hypothesis that "the average is lower than the value of m". In case of bilateral test, acceptance of null hypothesis means acceptance of product as compliant.

The inference result in the software depends on the sample size as the inference is based on confidence range. The problem of sample size influencing the confidence range and, as a consequence, the precision and reliability in such tests is presented below.

Importance of sample size during verification of a statistical hypothesis

The confidence range is the range determined on the basis of sample testing that covers, with a determined probability, the true, unknown value of population parameter. The probability of an event that the confidence range would cover the unknown value of the assessed parameter is known as confidence level. Usually, in case of the same sample size the length of confidence range increases with an increase of confidence level while in case of a determined confidence level that range is shortened as a consequence of increasing the sample size.

In practice the most frequently assumed α=0.05 and the confidence level is 1−α=0.95 [Sobczyk, 2004].

Graph 1 presents the dependence of confidence half range on sample size in the investigation carried out using the homogenized cheese. The individual asymptotic values converge to the average value $x_{śr}$.

On the basis of graph analysis it can be determined that in case of small sample sizes the average is contained in relatively large half-ranges of confidence, which means low precision of estimation of the population parameter. The larger the sample size the smaller the range within which the real average is contained and the larger the measurement precision. In practical terms the sample size proves to be highly important while its selection is always a problem, as it is impossible to define clearly at which sample size the satisfactory precision of $x_{śr}$ average estimation is achieved.

Precision of estimation based on the example of 2% fat milk

Using the application designed, an analysis was carried out to determine at which sample size further testing aimed at establishing compliance of the product with the standard could be abandoned (Table 1).

TABLE 1. Results of Student t-test of the difference between the average and value m (dm^3) for drinking milk with 2% fat content.

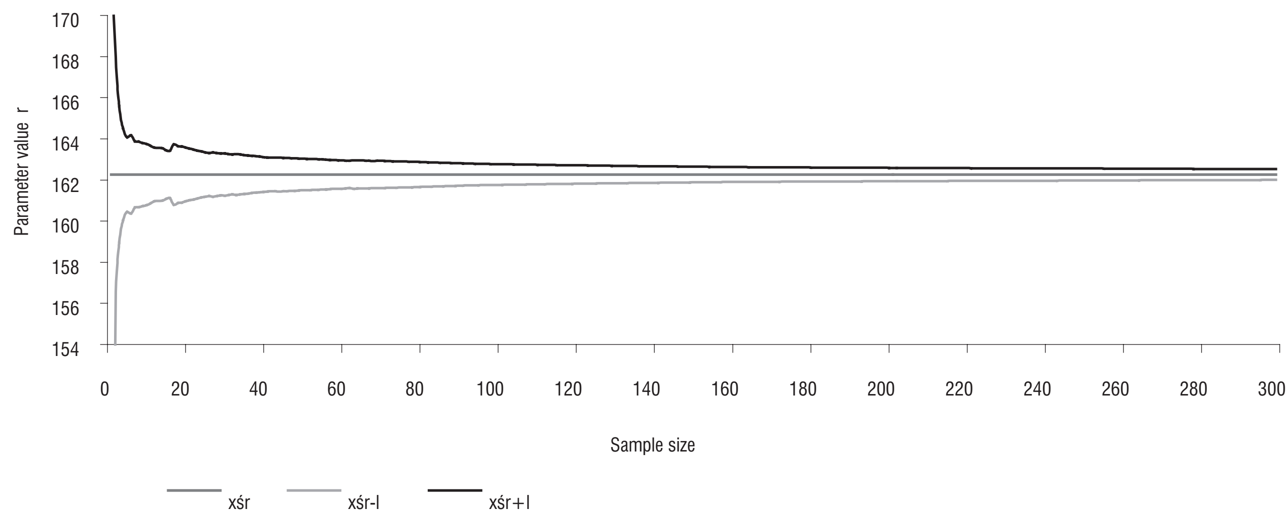
H_0: $x_{śr}$<m	The average value is lower than m			
H_1: $x_{śr}$≥m	The average is equal or higher than m			
			α=	0.05
Sure value m=	1.000	1.000	1.000	1.000
Symbol	MLEKO2%	MLEKO2%	MLEKO2%	MLEKO2%
$x_{śr}$=	1.00664	1.00643	1.00772	1.00682
$l_{x(1-a)}$=±	0.02021	0.01025	0.00871	0.00208
S(x) =	0.01628	0.01696	0.01509	0.01826
S^2(x) =	0.00027	0.00029	0.00023	0.00033
N =	5	13	14	300
Compared	1 with m	2 with m	3 with m	4 with m
$t_{obl.}$ =	0.91230	1.36773	1.91413	6.4638
P(t) =	0.20661	0.09823	0.03894	0.0000000002
$t_{a;f}$ =	2.13185	1.78229	1.77093	1.64996
H_0:	Accept	Accept	Reject	Reject

The analysis was carried out on the example of drinking milk at a confidence level of α=0.05.

The sure value equal to the declared value m=1 dm^3 was assumed. Analyzing different sample sizes it was established that already at sample size of 14 the null hypothesis was rejected meaning that the product was considered compliant with the declared specification. That procedure is very important, for example during packaging machine setup as it allows shortening the time required for selection of the optimum parameters.

Analysis of selection of the optimum deviation from the declared specification based on the example of the homogenized cheese

The analysis was carried out on the basis of the results obtained from measurements of cheese mass on the sample size n=300.



GRAPH 1. Relation between the size of confidence half ranges and the sample size on the example of homogenised cheese.

Using the application developed, assuming that the null hypothesis is true, it was determined at which suggested value chosen for the sure value m, the product could be considered compliant with the declared specification. That procedure was applied to the declared value that possessed no determined deviation.

Suggesting the average value, the optimum deviation from the declared value was determined as well (Table 2).

Cheese mass of 150 g consistent with the declared value was assumed for the mean value $x_{śr}$. By selecting the appropriate sure value m it was established that the difference between the value established in the specification (150 g) and the assumed standard values (149.74 and 150.26), for which the null hypothesis was accepted, was ± 0.26. Exceeding that value by ±0.27 resulted in rejection of the null hypothesis, *i.e*. considering the product noncompliant with the specification.

Establishment of the specific limit for deviations from the declared value is particularly important for enterprises that have not yet considered the acceptable limits for which the product is still considered compliant in their enterprise standards.

Estimation of the financial loss resulting from deviation of real product mass or volume from the declared value

An attempt to estimate the financial losses of the enterprise was made on the basis of simulated data presented in Table 3.

The day production in units, number of production days per month, monthly production and rough producer unit price were assumed as simulated values. The analysis was carried out on the basis of real averages.

As a result of the above analysis it was established that the enterprise loss per year accounts for PLN 110 352.16 as a consequence of inappropriate control of product quantity compliance with the declared value. The surplus of product units does not reflect the nature of the problem, however, in case of analyzing the simulated volume of produced batch, already in case of one-month period the financial losses that can be incurred by the enterprise become visible.

That analysis may serve as an additional component in enterprise management as it allows simple determination of losses resulting from inappropriate mass or volume of product per unit of production.

It should be highlighted how important observation of values declared in products' specification is for the enterprise – not only from the point of view of care for consumers' interest but also in the aspect of profitability of enterprise activity.

TABLE 3. Results of the analysis of projected financial losses incurred by an enterprise.

	PRODUCT	
	CHEESE (g)	MILK (dm^3)
Estimated day production in units	10 000	5 000
Number of production days per month	23	30
Monthly production	230 000	150 000
Average net mass or volume of a product	154.3866	1.0068
Mass or volume of a product acc. to specification	150	1
Exceeding the standard in percent	2.92%	0.68%
Real production of a month	35 508 918.00	151 022.43
Assumed production of a month acc. to specification	34 500 000	150 000
Difference between real production and that acc. to specification	1 008 918.00	1 022.43
Monthly loss in units	6 726	1 022
Rough unit price	1.20 PLN	1.10 PLN
Monthly financial loss	8 071.34 PLN	1 124.67 PLN
Annual financial loss	96 856.13 PLN	13 496.03 PLN

CONCLUSIONS

1. On the basis of hypotheses verification with Student t-test, an application was developed that allows assessing product batch compliance with specification requirements of individual products. Determination of highly precise values at which the product is considered compliant with the declared values enables estimation and minimization of

TABLE 2. Results of Student t-test of the difference between the average and m value for homogenised cheese.

H_0: $x_{śr}<m$	The average value is lower than m				
H_1: $x_{śr}\geq m$	The average is equal or higher than m				
					$\alpha=0.05$
Sure value m=	149.73	149.74	150.00	150.26	150.27
Symbol	TOSMAK	TOSMAK	TOSMAK	TOSMAK	TOSMAK
$x_{śr}$=	150	150	150	150	150
$l_{x(1-a)}=\pm$	0.26298	0.26298	0.26298	0.26298	0.26298
s(x) =	2.31456	2.31456	2.31456	2.31456	2.31456
$s^2(x)$ =	5.35718	5.35718	5.35718	5.35718	5.35718
n =	300	300	300	300	300
Compared	1 with m	2 with m	3 with m	4 with m	5 with m
$t_{obl.}$ =	2.02049	1.94566	0.00000	1.94566	2.02049
P(t) =	0.04422	0.05263	1.00000	0.05263	0.04422
$t_{a;f}$ =	1.96793	1.96793	1.96793	1.96793	1.96793
H_0:	Reject	Accept	Accept	Accept	Reject

losses resulting from inappropriate mass or capacity of a product in a unit package.

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MOŻLIWOŚCI WYKORZYSTANIA TESTÓW PARAMETRYCZNYCH W KONTROLI PRODUKCJI

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Celem pracy było opracowanie w środowisku arkusza kalkulacyjnego aplikacji do oceny, metodą testów parametrycznych, zgodności partii produktu z wartościami podanymi w specyfikacji artykułu.

Badania przeprowadzono na partiach wybranych produktów liczących 300 sztuk opakowanego wyrobu. Do weryfikacji uzyskanych danych zastosowano test Q – Dixona w celu eliminacji danych obarczonych błędem grubym. Jako metodę weryfikacji hipotez statystycznych zastosowano test istotności t przy postawionej hipotezie zerowej H_0: $x_{śr}>m$ wobec hipotezy alternatywnej H_1: $x_{śr}\leq m$.

Wnioskowanie w zastosowanych testach jest oparte na przedziale ufności i jest uzależnione od liczebności próbki. Liczebność próbki wpływa na wielkość przedziału ufności a przez to na dokładność i wiarygodność wnioskowania. Zbudowany program umożliwia przeprowadzenie analizy mającej na celu określenie, przy jakiej minimalnej liczebności próbki można już skończyć dalsze badanie, by ustalić zgodność wyrobu z normą. Ustalenie konkretnej granicy odchyleń od wartości deklarowanej w specyfikacji producenta ma szczególne znaczenie dla przedsiębiorstw, które w swoich normach zakładowych nie uwzględniły jeszcze dopuszczalnej granicy, przy której produkt ciągle uznawany jest za zgodny. Opracowana aplikacja umożliwia także oszacowanie strat finansowych ponoszonych przez przedsiębiorstwo, w wyniku nieprawidłowej kontroli zgodności ilości produktu z wartością deklarowaną.